Variable Smoothing in Bayesian Intrinsic Autoregressions

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Abstract: This poster discusses an extension to the Markov random field (MRF) defined in Besag et al. (1991) with the aim of providing a more flexible method for modelling spatial correlation with Bayesian spatial analysis.

Introduction

We propose an extension to the MRF for Bayesian spatial modelling of Besag et al. (1991) (BYM) by allowing the amount of smoothing to vary. In different parts of a map, area-specific smoothing parameters are defined; these are related to the variance of the MRF. We have tested our method on both simulated and real data sets; the new scheme is found to improve modelling of both slowly-varying levels of smoothness and discontinuities in the response surface.

Motivating Data

The Macaulay Institute surveys estates in Scotland to record the level of grazing and trampling impact. Spatial regression models have been used to smooth the data, but is a constant smoothing level appropriate? What about feeding stations, landscape features, etc?

Variable Smoothing Scheme

Our scheme is based upon a zero-centred CAR specification for a vector of univariate random variables \(u = (u_1, u_2, \ldots, u_n)^T\). This provides an intrinsic form for the prior of spatial random effect terms:

\[u_i | \{ u_i, \phi \} \sim N \left( \sum b_i u_i, \phi_i \right)\]

where

\[b_i = \frac{\lambda_i \phi_i}{\lambda_i + \phi_i} \times \left\{ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_j + \phi_i} \right\}^{-1}, \quad \phi_i = \left\{ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_j + \phi_i} \right\}^{-1}.

and where the \(\lambda\) are small-area specific precision contributions—we get one of these from each neighbour-pair.

While the prior form proposed is improper, the propriety of posterior distributions can be established by appeal to the proof of Theorem 2 in Ghosh et al. (1998).

Implementation

A full analysis using our MRF scheme consists of two stages. We first run a standard BYM analysis, from which we gain estimates of the spatial random effects. These are then used to construct informative priors for the \(\lambda\) parameters of our variable smoothing model.

WinBUGS (Lunn et al., 2000) can be used for both stages, although the expressions for the new MRF have to be coded long-hand in WinBUGS.

Simulation Study

We consider two types of simulated data—one having an west-to-east trend in random effect variance, and another having “patches” of generally higher values, representing discontinuities in the landscape. We have 20 sets of each type.

Application

The map of random effect differences highlights influence of human interventions; that of local variance contributions shows how the smoothing is adapted. These data were first analysed in Brewer et al. (2004).

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References


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