Mathematical modelling

B. Marshall

Models are an intricate part of our everyday life. We use them to make decisions in the belief that taking certain actions will probably result in particular outcomes. The underlying models are simplified but effective "pictures" of reality. Likewise, models are the cornerstone of scientific method. A model is essentially a hypothesis, a statement of how something works or interacts, which is tested by its predictions. Its truth can never be proven, since it can always fail the next test. Newtonian Mechanics was believed to apply in all circumstances, but ultimately failed in conditions extreme compared to our everyday experience. Nevertheless, much of our modern world still exploits the models of classical mechanics because the predictions remain accurate in the world we live in.

We use the familiar to describe the unfamiliar. We define things by their behaviour and by analogy. The electron with a fixed mass and charge, could be thought of as a minute billiard ball. These properties are exploited in the domestic TV. However, it also exhibits wave like properties. Imagine waves entering a harbour. They emerge within as circular wave fronts and with two entrances, close by, produce the familiar patterns of interference. Exactly the same patterns were seen when a single electron was "aimed" at a much smaller pair of slits. This means the electron "passed" through both slits at the same time, which seems surprising if you only think of electrons as particles. Electrons are both like particles and like waves.

Aristotle was a keen observer, especially of biological systems, and emphasised the importance of considering the whole. He believed the "physical method", promoted by Democritus, failed because it tried to explain things by decomposing them into their parts and ignored the whole. Knowing that a house is built of bricks, mortar and timber tells us little about its architecture and nothing of its purpose. Similarly, reducing an animal or plant to its parts and ignoring the whole tells us nothing of its form or function. Aristotle believed that the study of nature should focus on the coordination of the parts in the whole and this is never more applicable than today. The behaviour of the system as a whole is referred to as an emergent property. It is not a property of the component parts but a consequence of the interactions between them. The Gas Laws are a classic example – the temperature, pressure and volume of a gas are related by a simple rule. Boyle discovered this rule, an emergent property, by making measurements at the scale of the system. If he had been able to break down the gas into its components, to see the molecules in-situ, then he would have observed them moving in random directions, at different speeds and occasionally colliding (interacting) with each other and the walls of the container. He would have seen nothing of the large scale behaviour of gases. Only by modelling these interactions could he then possibly predict the emergent property. In ecological systems we face this challenge. It is easier to observe the individual than

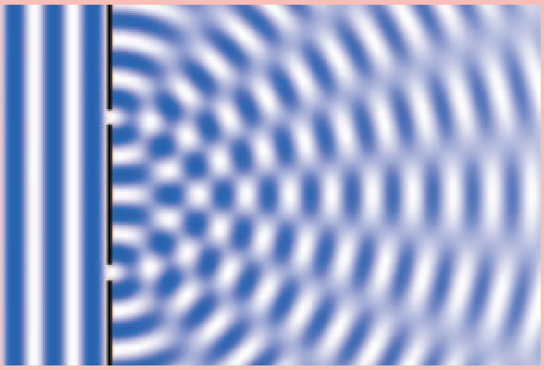

Figure 1 An electron is both like a particle and a wave. A single electron passes through both slits at the same time producing interfering waves on the far side, like waves of the sea passing through two entrances to a harbour.

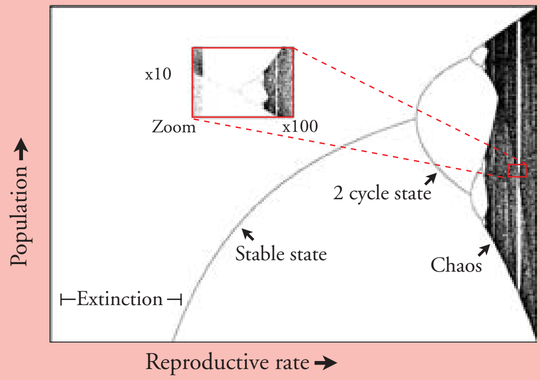

Figure 2 Steps to chaos. The behaviour of a population with a limited food resource can change dramatically if the reproductive rate of the individuals is altered e.g. a different species or warmer temperatures.

Figure 3 Frost on a windscreen. Simple mechanisms can create rich, life like patterns.

the whole. If we are to predict the future survival of an ecosystem accurately then we must capture the key interactions between individuals.

Much of nature is non-linear and yet much of mathematics is dedicated to linear problems, because analytical solutions can usually be found to the linear problems but rarely to the former. The power of computers has enabled us to explore non-linear systems in much greater depth. Rich and complex behaviour can emerge from the simplest of non-linear systems. An example of this is the chaotic behaviour that can arise from a simple but powerful ecological model of population growth. The population grows in proportion to its size, initially doubling in each life cycle. However, the population as a whole has finite food supply. As the population expands, food becomes limited and rates of mortality exceed those of birth. The behaviour of this model is critically dependent on the reproductive potential of the individual. For low values, mortality always exceed birth rates and the population becomes extinct. For moderate values, stable population densities are reached. At higher rates, the behaviour changes dramatically from a single stable population to cycling through sets of fixed densities: first one bifurcation, where the population oscillates between two densities on alternate life cycles, and then more bifurcations. Each doubling requiring smaller and smaller increases in reproductive potential until the system becomes "chaotic". In chaos, the population is constantly changing at each life cycle and never revisits an earlier density. Although chaotic, the range of densities is restricted and some regions are more likely to be visited than others. The behaviour seems random but is not. In principle, if one knew the population precisely at one point in time then one could predict all future populations. The problem is that the smallest error in this estimate is rapidly magnified over a few life cycles and predictability lost. This behaviour is an intrinsic property of the system and not caused by any external, fluctuating influences. Weather exhibits similar changes in behaviour, moving from relatively stable states where long term-forecasts are reliable to more turbulent, chaotic states where forecasts are limited to a day or two and with much less confidence.

Models are simplified representations of reality with the ability to predict. We often use the familiar to describe the unfamiliar, but must be wary of any preconceptions and assumptions made. It is important to consider the whole, how the components interact, in order to discover the emergent properties of a system. We must be aware that even the simplest biological systems can exhibit a rich behaviour – an apparent population crash may be an intrinsic property of the system rather than an indication of an external, adverse influence. We aim to combine mathematical modeling with biological experimentation while keeping an open mind when studying systems.